Interaction or Correlation? An analysis of Choquet integral approach in MCDA real situations

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Abstract. Choquet integral as a decision model in MCDA is based on the use of a capacity, i.e. a non-additive set function able to represent interactions between criteria. When facing an elicitation process, the interpretation of a capacity in terms of interaction between criteria can be questionable as the elicitated capacity depends on the data, which can be correlated or not. We propose here an ANOVA-based index that aims at determine if the elicitated capacity can be significantly interpreted in terms of criteria interaction or not.

1 Introduction

When facing a new multicriteria decision aiding (MCDA) problem, experts suggest the use of a specific MCDA model, and try to found the model parameters that best fit the situation and decision taken by the Decision Maker (DM). This phase is called "parameters elicitation". In the specific case of using a Choquet integral as preference modelling, the elicitation process consists in finding weights for each criterion and each subset of the criteria set. Weights of subsets with cardinal greater than or equal to two can be seen as an indication of the possible interaction between criteria and should be interpreted in a natural meaning for the DM. Therefore it is crucial to determine whether these interactions are really ones or just appear as artefacts due to the structure of the dataset used to elicitate the model parameters. Our main issue in this paper is to propose a robustness index and a specific process that aim at determine if the detected interactions during the elicitation process are significant or not.

Choquet integral (see [2, 4, 5] for a brief review) is based on the use of a capacity. Let P be the set of criteria.

Definition 1. *A capacity is a set function* v: $2^{\mathcal{P}} \to [0,1]$ *such that:*

- $v(\emptyset) = 0$, $v(\mathcal{P}) = 1$ *(boundary conditions),*
- $\forall \mathcal{A}, \mathcal{B} \in 2^{\mathcal{P}}$ *such that* $\mathcal{A} \subseteq \mathcal{B}, v(\mathcal{A}) \leq v(\mathcal{B})$ *(monotonicity).*

Any set function v: $2^{\mathcal{P}} \to [0,1]$ can be uniquely expressed in terms of its Moebius representation by:

$$v(\mathcal{A}) = \sum_{\mathcal{B} \subseteq \mathcal{A}} m_v(\mathcal{B}) \ \ \forall \mathcal{A} \subseteq \mathcal{P},$$

where the set function $m_v : 2^{\mathcal{P}} \to \mathbb{R}$ is called the Moebius transform or Moebius representation of v and is given by

$$m_v(\mathcal{A}) = \sum_{\mathcal{B} \subseteq \mathcal{A}} (-1)^{(a-b)} v(\mathcal{B}) \ \ \forall \mathcal{A} \subseteq \mathcal{P},$$

where a and b are the cardinals of $\mathcal{A}$ and $\mathcal{B}$. The Moebius coefficients can be interpreted as interaction index between criteria. A negative Moebius index for a subset $\mathcal{A}$ means that the criteria in $\mathcal{A}$ have a negative interaction, i.e. that the importance of the criteria subset $\mathcal{A}$ is less than the sum of the importance of each criterion included in $\mathcal{A}$. A positive Moebius index for a subset $\mathcal{A}$ means that the criteria in $\mathcal{A}$ have a positive interaction, i.e. that the importance of the criteria subset $\mathcal{A}$ is more than the sum of the importance of each criterion included in $\mathcal{A}$. Several elicitation process have been proposed to obtain a capacity that model a specific multicriteria decision situation. Such a process needs as input:

- a set X including m alternatives described on p criteria (or variables) with values included in the same value set $[0; a]$. We can consider, without any loss of generality, that these values are all included in the interval $[0; 1]$,
- for each alternative $x \in X$ a global score $s(x) \in [0; 1]$.

An elicitation process is an algorithm which produces a capacity v such that the difference between $s(x)$ and $f_v^C(x)$ is minimized. Several elicitation processes have been proposed with various minimizing functions, see [3] for a review. Whatever the process, we can suppose now that we obtained a capacity v from X and $\{s(x), x \in X\}$. This capacity v should be interpreted in terms of positive and negative interactions between criteria. The question then is: does this interpretation is robust with respect to the given data?

2 Robustness index definition

We propose in this section an index to determine if the interaction between criteria that have been obtained through the elicitation process are significant or not. The main idea is to test if different capacities lead to different ranks for the alternatives. If each capacity gives a different ranking on X, it means that a specific capacity, obtained by elicitation from specific scores, is really meaningful in terms of interactions. If the ranking on X is almost the same whatever the capacity is, it means that it isn't possible to give an interpretation of the interactions between criteria, as contradictory interactions can conclude to the same ranking. We propose a global index to measure the robustness of a ranking with respect to a change of the aggregation function parameters. This index is based on a more general one detailed in a previous work [6]. The Rank Robustness Index (RRI) is based on an analysis of variance approach. The ranking on $\mathcal{X}$ depends on both:

1. the values taken by each individual x on each variable $i \in \{1, \ldots, p\}$
2. the capacity v for the Choquet integral.

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A score obtained via a Choquet integral with capacity v is given to each alternative. A ranking R on X is then obtained based on these scores. So a ranking on the m individuals of X is obtained for any capacity v. Therefore n different capacities lead to n (possibly) different rankings. Capacity influence on the rank of an individual x can be measured by the intrinsic variance of the individual rankings: the lowest the variance, the less influence have the capacity on the ranking. We focus on the Sum of Squared Deviations (SSD) of individual rankings. The standard analysis of variance theory (ANOVA) says that SSD can be split into two factors: the SSD due to the capacities and the SSD only due to the values taken by each individual on each variable (the data);

$$SSD_{total} \quad = \quad SSD_{cap} + SSD_{data}. \qquad (1)$$

The global SSD can easily be computed and is equal to

$$\frac{n(m-1)m(m+1)}{12}.$$

The SSD due to the capacities is equal to $\sum_{i=1}^{m}\sum_{j=1}^{n}(r_{ij}-r_{i.})^2$ where r_{ij} is the rank of individual i in the ranking obtained with capacity j and $r_{i.}$ is the mean rank of individual i. SSD_{cap} is exactly equal to $n\sum_{i=1}^{m} var_i$ where var_i is the variance of individual i ranks.

The SSD due to the data is equal to $n\sum_{i=1}^{m}(r_{i.}-r_{..})^2$ where $r_{i.}$ is the mean rank of individual i and $r_{..}$ the global mean rank (equal to $(m+1)/2$).

As in usual ANOVA, we can compute the percentage of variance that is explained by the data versus the capacities. Therefore we define RRI as the ratio between the SSD depending on the data and the global SSD:

$$RRI = \frac{SSD_{data}}{SSD_{total}} = 1 - \frac{SSD_{cap}}{SSD_{total}} = 1 - \frac{12\sum_{i=1}^{m} var_i}{m(m^2-1)}. \quad (2)$$

As far as var_i are known, RRI does not depend on n. We develop below an estimation process of RRI using n capacities.

RRI can be interpreted in terms of support to the interaction between criteria hypothesis:

- if $RRI = 0$ then it means that all the information is in the capacity. Then we can't change the capacity without influencing the ranking, and therefore it means that the elicitated capacity are specific to the situation. Then the Moebius coefficients of the elicitated capacity can be interpreted in terms of real interactions between criteria.
- if $RRI = 1$ it means that all the information is in the data : whatever the capacity, the ranking will be always the same. therefore interactions have no meaning as opposite interactions lead to the same ranking.
- so the closer to 0 is the RRI of a situation, the more accurate is the interpretation of the capacity in terms of interaction.

3 Robustness index computation

The set of possible capacities is supposed to be infinite. It is then almost impossible to compute exact values of var_i, and so the value of RRI directly. Therefore, we have to determine an estimation of RRI. We propose to use a Monte-Carlo method to do so:

1. Generate n different capacities randomly in the space of parameters vectors. We use algorithm described in [1]. Then use these n capacities to obtain n different scores vectors and so n different rankings on the set of m individuals.
2. Compute the exact rank variance of each individual var_i^* of the n sampled rankings obtained in previous step.
3. Compute RRI as one minus the ratio of the mean of individual variances divided by the exact total variance:

$$\widehat{RRI} = 1 - \frac{12\sum_{i=1}^{m} var_i^*}{m(m^2-1)}.$$

4. It is possible to repeat the point estimation to estimate also the variance of the estimator, and then to obtain a confidence interval of RRI. If m is the average value of n RRI estimations and s^2 is the estimated variance of n RRI estimations, then the confidence interval of RRI estimation is $]m - zs/\sqrt{n}; m + zs/\sqrt{n}[$, with z the normal corresponding value of the chosen quantile (typically z=1.96 for a 95% confidence interval).

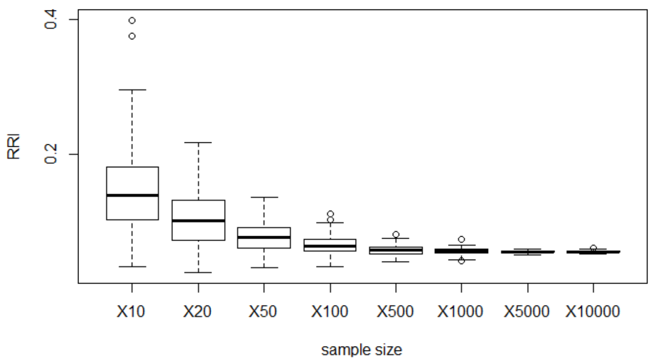

Figure 1. Estimations of RRI - canonical example with 5 criteria and 10 alternatives [3]

Experiments show that a precise estimation of RRI can be obtained with a small number of capacities. Typically, simulation of 500 capacities, or even only 100, leads to estimations of RRI good enough to see if the ranking is robust or not to a change of capacity. An experiment on the data proposed in [3] shows that RRI is close to 0, which means that the elicitated capacity is really meaningful in terms of criteria interaction. It shows also that precise estimation of RRI is obtained even if the capacities sample number is low (around 100).

REFERENCES

[1] I. Díaz E.F. Combarro and P. Miranda, 'On random generation of fuzzy measures', *Fuzzy Sets and Systems, Special issue on AGOP 2011 and EUSFLAT/LFA 2011*, **228**(4), 64–77, (2013).

[2] M. Grabisch, 'The application of fuzzy integrals in multicriteria decision making', *EJOR*, **89**, 445–456, (1996).

[3] M. Grabisch, I. Kojadinovic, and P. Meyer, 'A review of methods for capacity identification in choquet integral based multi-attribute utility theory: Applications of the kappalab R package', *European Journal of Operational Research*, **186**(2), 766–785, (2008).

[4] M. Grabisch and C. Labreuche, 'A decade of application of the choquet and sugeno integrals in multi-criteria decision aid', *Annals OR*, **175**(1), 247–286, (2010).

[5] M. Grabisch, J-C. Marichal, R. Mesiar, and E. Pap, *Aggregation functions*, volume 127 of *Encyclopedia of Mathematics and its Applications*, Cambridge University Press, Cambridge, UK, 2009.

[6] A. Rolland and J. Cugliari, 'Ranking and top-k ranking robustness index to measure dependence to parameters'. working paper or preprint, June 2018.