2-additive Choquet Optimal Solutions in Multiobjective Optimization Problems

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Abstract. In this paper, we propose a sufficient condition for a solution to be optimal for a 2-additive Choquet integral in the context of multiobjective combinatorial optimization problems. A 2-additive Choquet optimal solution is a solution that optimizes at least one set of parameters of the 2-additive Choquet integral. We also present a method to generate 2-additive Choquet optimal solutions of multiobjective combinatorial optimization problems. The method is experimented on some Pareto fronts and the results are analyzed.

Keywords: Choquet integral, Multicriteria decision making, Fuzzy measure, Multiobjective combinatorial optimization, k-additivity.

1 Introduction

Multiobjective combinatorial optimization problems (MOCO) aim at finding the Pareto optimal solutions among a combinatorial set of feasible solutions. A Pareto optimal solution is a solution that is not Pareto dominated by any other solutions; the set of all these solutions is named the Pareto optimal set (or the Pareto front, in the objective space). However, the set of all the Pareto optimal solutions can be huge, especially in the case of several objectives [1]. Therefore it is worth to study the set of solutions that optimize a specific function, for example a weighted sum, as it generally reduces the size of the set of interesting Pareto optimal solutions. In this latter case, it is well-known that the set of potential optimal solutions is the convex envelop of the feasible solutions set. In order to attain solutions located in the non-convex part of the feasible solutions set, other aggregation operators could be used as function to be optimized. In this paper, we will focus on a specific aggregation operator: the Choquet integral.

The Choquet integral [2] is one of the most powerful tools in multicriteria decision making [3, 4]. A Choquet integral can be seen as an integral on a non-additive measure (or capacity or fuzzy measure), that is an aggregation operator that can model interactions between criteria. It presents extremely wide expressive capabilities and can model many specific aggregation operators, including, but not limited to, the weighted sum, the minimum, the maximum, all

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the statistic quantiles, the ordered weighted averaging operator [5], the weighted ordered weighted averaging operator [6], etc.

However, this high expressiveness capability needs a great number of parameters. While a weighted sum operator with p criteria requires $p-1$ parameters, the definition of the Choquet integral with p criteria requires the setting of 2^p-2 values, which can be high even for low values of p. The notion of k-additivity has been introduced by Grabisch [7] in order to reduce the number of needed parameters while keeping the possibility to take into account the interactions between k criteria among the p criteria; typically for $k=2$, one only needs $\frac{p^2+p}{2}$ parameters.

Some papers already deal with the optimization of the Choquet integral of MOCO problems [8–10] when the Choquet integral is completely defined by the decision maker. Recently, Lust and Rolland investigated a method to generate the whole set of Choquet optimal solutions. The aim is to compute all the solutions that are potentially optimal for at least one parameter set of the Choquet integral. This method was studied in the particular case of biobjective combinatorial optimization problems [11], and for the general case in [12]. A characterization of the Choquet optimal solutions through a set of weighted-sum optimal solutions has been stated.

In this contribution, we focus on the specific case of the 2-additive Choquet integral. In the next section, we recall the definition of the Choquet integral. We propose then a sufficient condition for a solution to be Choquet optimal with a 2-additive capacity. We finally present some experimental results where we study the difference between the exact set of Choquet optimal with a 2-additive capacity and the set obtained with the sufficient condition proposed in this paper.

2 Aggregation Operators and Choquet Integral

We introduce in this section the basic concepts linked to multiobjective combinatorial optimization problems, the weighted sum and the Choquet integral.

2.1 Multiobjective Combinatorial Optimization Problems

A multiobjective (linear) combinatorial optimization (MOCO) problem is generally defined as follows:

$$
\begin{aligned}
&\text{“}\max_{x}\text{”}\, f(x) = Cx = (f_1(x), f_2(x), \ldots, f_p(x)) \\
&\text{subject to } Ax \leq b \\
&\qquad\qquad x \in \{0,1\}^n
\end{aligned}
$$

$$
\begin{array}{ccl}
x \in \{0,1\}^n & \longrightarrow & n \text{ variables} \\
C \in \mathbb{R}^{p \times n} & \longrightarrow & p \text{ objective functions} \\
A \in \mathbb{R}^{r \times n} \text{ and } b \in \mathbb{R}^{r \times 1} & \longrightarrow & r \text{ constraints}
\end{array}
$$

A feasible solution x is a vector of n variables, having to satisfy the r constraints of the problem. Therefore, the feasible set in decision space is given by $\mathcal{X} = \{x \in \{0,1\}^n : Ax \leq b\}$. The image of the feasible set is given by $\mathcal{Y} = f(\mathcal{X}) = \{f(x) : x \in \mathcal{X}\} \subset \mathbb{R}^p$. An element of the set $\mathcal{Y}$ is called a cost-vector or a point.

Let us consider in the following, without loss of generality, that all the objectives have to be maximized and we design by $\mathcal{P}$ the set of objectives $\{1,\ldots,p\}$. The Pareto dominance relation (P-dominance for short) is defined, for all $y^1, y^2 \in \mathbb{R}^p$, by:

$$y^1 \succ_P y^2 \iff [\forall k \in \mathcal{P}, y^1_k \geq y^2_k \text{ and } y^1 \neq y^2]$$

The strict Pareto dominance relation (sP-dominance for short) is defined as follows:

$$y^1 \succ_{sP} y^2 \iff [\forall k \in \mathcal{P}, y^1_k > y^2_k]$$

Within a feasible set $\mathcal{X}$, any element x^1 is said to be *P-dominated* when $f(x^2) \succ_P f(x^1)$ for some x^2 in $\mathcal{X}$, *P-optimal* (or *P-efficient*) if there is no $x^2 \in \mathcal{X}$ such that $f(x^2) \succ_P f(x^1)$ and *weakly P-optimal* if there is no $x^2 \in \mathcal{X}$ such that $f(x^2) \succ_{sP} f(x^1)$. The *P-optimal set* denoted by $\mathcal{X}_P$ contains all the P-optimal solutions. The image $f(x)$ in the objective space of a P-optimal solution x is called a P-non-dominated point. The image of the P-optimal set in $\mathcal{Y}$, equal to $f(\mathcal{X}_P)$, is called the Pareto front, and is denoted by $\mathcal{Y}_P$.

2.2 Weighted Sum

Instead of generating the *P-optimal set*, one can generate the solutions that optimize an aggregation operator. One of the most popular aggregation operator is the weighted sum (WS), where non-negative importance weights $\lambda_i (i = 1,\ldots,p)$ are allocated to the objectives.

Definition 1. *Given a vector $y \in \mathbb{R}^p$ and a weight set $\lambda \in \mathbb{R}^p$ (with $\lambda_i \geq 0$ and $\sum_{i=1}^p \lambda_i = 1$), the WS $f^{ws}_\lambda(y)$ of y is equal to:*

$$f^{ws}_\lambda(y) = \sum_{i=1}^{p} \lambda_i y_i$$

Definition 2. *Let $x \in \mathcal{X}$ and $y = f(x)$ be its image in $\mathcal{Y}$. If $\exists\, \lambda \in \mathbb{R}^p_+$ ($\lambda_i > 0$) such that $f^{ws}_\lambda(y) \geq f^{ws}_\lambda(y^2)\ \forall\, y^2 \in \mathcal{Y}$ then x is a* supported *P-optimal solution, and its image y a supported P-non-dominated point.*

Note that there exist P-optimal solutions that do not optimize a WS, and they are generally called *non-supported* P-optimal solutions [1].

2.3 Choquet Integral

The Choquet integral has been introduced by Choquet [2] in 1953 and has been intensively studied, especially in the field of multicriteria decision analysis, by several authors (see [3, 4, 13] for a brief review).

We first define the notion of capacity, on which the Choquet integral is based.

Definition 3. *A capacity is a set function* v: $2^{\mathcal{P}} \rightarrow [0,1]$ *such that:*

- $v(\emptyset) = 0$, $v(\mathcal{P}) = 1$ *(boundary conditions)*
- $\forall \mathcal{A}, \mathcal{B} \in 2^{\mathcal{P}}$ *such that* $\mathcal{A} \subseteq \mathcal{B}, v(\mathcal{A}) \leq v(\mathcal{B})$ *(monotonicity conditions)*

Therefore, for each subset of objectives $\mathcal{A} \subseteq \mathcal{P}$, $v(\mathcal{A})$ represents the importance of the coalition $\mathcal{A}$.

Definition 4. *The Choquet integral of a vector* $y \in \mathbb{R}^p$ *with respect to a capacity* v *is defined by:*

$$f_v^C(y) = \sum_{i=1}^{p} \big(v(Y_i^{\uparrow}) - v(Y_{i+1}^{\uparrow})\big) y_i^{\uparrow}$$
$$= \sum_{i=1}^{p} (y_i^{\uparrow} - y_{i-1}^{\uparrow}) v(Y_i^{\uparrow})$$

where $y^{\uparrow} = (y_1^{\uparrow}, \ldots, y_p^{\uparrow})$ *is a permutation of the components of* y *such that* $0 = y_0^{\uparrow} \leq y_1^{\uparrow} \leq \ldots \leq y_p^{\uparrow}$ *and* $Y_i^{\uparrow} = \{j \in \mathcal{P}, y_j \geq y_i^{\uparrow}\} = \{i^{\uparrow}, (i+1)^{\uparrow}, \ldots, p^{\uparrow}\}$ *for* $i \leq p$ *and* $Y_{(p+1)}^{\uparrow} = \emptyset$.

We can notice that the Choquet integral is an increasing function of its arguments.

We can also define the Choquet integral through the Möbius representation [14] of the capacity. Any set function v: $2^{\mathcal{P}} \rightarrow [0,1]$ can be uniquely expressed in terms of its Möbius representation by:

$$v(\mathcal{A}) = \sum_{\mathcal{B} \subseteq \mathcal{A}} m_v(\mathcal{B}) \quad \forall \mathcal{A} \subseteq \mathcal{P}$$

where the set function $m_v : 2^{\mathcal{P}} \rightarrow \mathbb{R}$ is called the Möbius transform or Möbius representation of v and is given by

$$m_v(\mathcal{A}) = \sum_{\mathcal{B} \subseteq \mathcal{A}} (-1)^{(a-b)} v(\mathcal{B}) \quad \forall \mathcal{A} \subseteq \mathcal{P}$$

where a and b are the cardinals of $\mathcal{A}$ and $\mathcal{B}$.

A set of 2^p coefficients $m_v(\mathcal{A})$ ($\mathcal{A} \subseteq \mathcal{P}$) corresponds to a capacity if it satisfies the boundary and monotonicity conditions [15]:

1. $m_v(\emptyset) = 0, \sum_{\mathcal{A} \subseteq \mathcal{P}} m_v(\mathcal{A}) = 1$
2. $\sum_{\mathcal{B} \subseteq \mathcal{A},\ i \in \mathcal{B}} m_v(\mathcal{B}) \geq 0 \quad \forall \mathcal{A} \subseteq \mathcal{P}, i \in \mathcal{P}$

We can now write the Choquet integral with the use of Möbius coefficients. The Choquet integral of a vector $y \in \mathbb{R}^p$ with respect to a capacity v is defined as follows:

$$f_v^C(y) = \sum_{\mathcal{A} \subseteq \mathcal{P}} m_v(\mathcal{A}) \min_{i \in \mathcal{A}} y_i$$

A Choquet integral is a versatile aggregation operator, as it can express preferences to a wider set of solutions than a weighted sum, through the use of a non-additive capacity. When solving a MOCO problem, with the Choquet integral, one can attain *non-supported* P-optimal solutions, while it is impossible with the weighted sum [11].

However, this model needs also a wider set of parameters to capture this non-additivity. For p criteria, one only needs $p-1$ weights to use a weighted sum, where 2^p-2 weights are needed to use a Choquet integral based on a capacity. Therefore, the concept of k-additivity has been introduced by [7] to find a compromise between the expressiveness of the model and the number of needed parameters.

Definition 5. *A capacity v is said to be k-additive if*

- $\forall \mathcal{A} \subseteq \mathcal{P}, m_v(\mathcal{A}) = 0$ *if* $card(\mathcal{A}) > k$
- $\exists\, \mathcal{A} \subseteq \mathcal{P}$ *such that* $card(\mathcal{A}) = k$ *and* $m_v(\mathcal{A}) \neq 0$

We will specially focus in this paper on 2-additive capacities and propose a sufficient condition for a solution of a MOCO problem to be 2-additive Choquet optimal.

3 Characterization of Choquet Optimal Solutions

3.1 Choquet Optimal Solutions

A characterization of the set of Choquet optimal solutions of a MOCO problem has been proposed in [12]. We briefly recall it here.

We denote the set of Choquet optimal solutions of a MOCO problem with p objectives $\mathcal{X}_C$: it contains at least one solution $x \in \mathcal{X}$ optimal for each possible Choquet integral, that is $\forall v \in \mathcal{V}, \exists\, x_c \in \mathcal{X}_C \,|\, f_v^C(f(x_c)) \geq f_v^C(f(x)) \;\forall x \in \mathcal{X}$, where $\mathcal{V}$ represents the set of capacity functions defined over p objectives. Note that each Choquet optimal solution is at least weakly P-optimal [11].

In [11], Lust and Rolland studied the particular case of two objectives and they showed that $\mathcal{X}_C$ could be obtained by generating all WS-optimal solutions in each subspace of the objectives separated by the bisector ($f_1(x) \geq f_2(x)$ or $f_2(x) \geq f_1(x)$), and by adding a particular point M with $M_1 = M_2 = \max_{x \in \mathcal{X}} \min(f_1(x), f_2(x))$.

In [12], Lust and Rolland extended this characterization to the general case.

Let σ be a permutation on $\mathcal{P}$. Let O_σ be the subset of points $y \in \mathbb{R}^p$ such that $y \in O_\sigma \iff y_{\sigma_1} \geq y_{\sigma_2} \geq \ldots \geq y_{\sigma_p}$.

Let p_{O_σ} be the following application:

$$p_{O_\sigma} : \mathbb{R}^p \to \mathbb{R}^p, (p_{O_\sigma}(y))_{\sigma_i} = (\min(y_{\sigma_1}, \ldots, y_{\sigma_i})), \forall i \in \mathcal{P}$$

For example, if $p = 3$, for the permutation (2,3,1), we have:

$$p_{O_\sigma}(y) = \big(\min(y_2, y_3, y_1), \min(y_2), \min(y_2, y_3)\big)$$

We denote by $\mathcal{P}_{O_\sigma}(\mathcal{Y})$ the set containing the points obtained by applying the application $p_{O_\sigma}(y)$ to all the points $y \in \mathcal{Y}$. As $(p_{O_\sigma}(y))_{\sigma_1} \geq (p_{O_\sigma}(y))_{\sigma_2} \geq \ldots \geq (p_{O_\sigma}(y))_{\sigma_p}$, we have $\mathcal{P}_{O_\sigma(\mathcal{Y})} \subseteq O_\sigma$.

Theorem 1

$$\mathcal{Y}_C \cap O_\sigma = \mathcal{Y} \cap WS(\mathcal{P}_{O_\sigma}(\mathcal{Y}))$$

where $WS(\mathcal{P}_{O_\sigma}(\mathcal{Y}))$ designs the set of WS-optimal points of the set $\mathcal{P}_{O_\sigma}(\mathcal{Y})$.

This theorem characterizes the solutions which can be Choquet optimal in the set of feasible solutions as being, in each subspace of the objective space $\mathcal{Y}$ where $y_{\sigma_1} \geq y_{\sigma_2} \geq \ldots \geq y_{\sigma_p}$, the solutions that have an image corresponding to a WS-optimal point in the space composed of the original subspace plus the projection of all the other points following the application p_{O_σ}.

Proof: see [12].

3.2 2-additive Choquet Optimal Solutions

We are now interested in the definition of the set of solutions of a MOCO problem that potentially optimize a 2-additive Choquet integral (and not a general Choquet integral). How does the constraints of 2-additivity restrict the set $\mathcal{Y}_C$? We will denote $\mathcal{Y}_{C^2}$ the set of 2-additive Choquet optimal solutions. As stated above, σ is a permutation on $\mathcal{P}$ and O_σ is the subset of points $y \in \mathbb{R}^p$ such that $y \in O_\sigma \iff y_{\sigma_1} \geq y_{\sigma_2} \geq \ldots \geq y_{\sigma_p}$.

Theorem 2

$$\forall \delta \in \Delta, \textit{if } y \in \mathcal{Y} \cap WS(\mathcal{P}^\delta_\sigma(\mathcal{Y})) \Rightarrow y \in \mathcal{Y}_{C^2} \cap O_\sigma$$

where:

- δ is an application $\mathcal{P} \to \mathcal{P}$ such that $\delta(1) = 1$ and $\delta(i) < i \ \forall i \neq 1$. Let Δ be the set of all applications δ.
- p^δ_σ is an application on $\mathcal{Y}$ such that $(p^\delta_\sigma(y))_{\sigma_i} = \min(y_{\sigma_{\delta(i)}}, y_{\sigma_i})$.
  For example, if $p = 4$, for the permutation (1,2,3,4) and $\delta = (1, 1, 2, 3)$, we have:
  - $(p^\delta_\sigma(y))_1 = \min(y_1, y_1) = y_1$
  - $(p^\delta_\sigma(y))_2 = \min(y_1, y_2)$
  - $(p^\delta_\sigma(y))_3 = \min(y_2, y_3)$
  - $(p^\delta_\sigma(y))_4 = \min(y_3, y_4)$
- $\mathcal{P}^\delta_\sigma(\mathcal{Y})$ is the set containing the points obtained by applying the application p^δ_σ to all the points $y \in \mathcal{Y}$.
- $WS(\mathcal{P}^\delta_\sigma(\mathcal{Y}))$ designs the set of supported points of the set $\mathcal{P}^\delta_\sigma(\mathcal{Y})$.

Proof

In the following, we will denote O_σ as simply O for the sake of simplicity, and we will consider, without loss of generality, that the permutation σ is equal to $(1, 2, \ldots, p)$, that is $y \in O \Leftrightarrow y_1 \geq y_2 \geq \cdots \geq y_p$. We will consequently note p^δ_σ as simply p^δ and $\mathcal{P}^\delta_\sigma(\mathcal{Y})$ as $\mathcal{P}^\delta(\mathcal{Y})$. We know that $\mathcal{Y}_{C^2} \subseteq \mathcal{Y}$ and then $\mathcal{Y}_{C^2} \cap O \subseteq \mathcal{Y} \cap O$.

Let us suppose that $y \in O$. Let $y \in WS(\mathcal{P}^\delta(\mathcal{Y})) \cap \mathcal{Y}$. Then there are $\lambda_1, \ldots, \lambda_p \geq 0$ such that $\sum_{i=1}^{p} \lambda_i = 1$ and

$$\forall z \in \mathcal{Y}, \ \sum_{i \in \mathcal{P}} \lambda_i y_i \geq \sum_{i \in \mathcal{P}} \lambda_i p^\delta(z)_i$$

By definition, $p^\delta(z)_i = \min(z_{\delta(i)}, z_i), \forall i \in \mathcal{P}$.

Let $\mathcal{A} \subseteq \mathcal{P}$. Let us define a set function m such that $m(\mathcal{A}) = \lambda_i$ if $\mathcal{A} = \{\delta(i), i\}$ and $m(\mathcal{A}) = 0$ if not.

Then

$$\begin{aligned}\sum_{i \in \mathcal{P}} \lambda_i (p^\delta(z))_i &= \sum_{i \in \mathcal{P}} \lambda_i \min(z_{\delta(i)}, z_i) \\ &= \sum_{\mathcal{A} \subseteq \mathcal{P}} m(\mathcal{A}) \min_{i \in \mathcal{A}} z_i\end{aligned}$$

Let us remind that the set function m corresponds to a capacity v if:

1. $m(\emptyset) = 0, \ \sum_{\mathcal{A} \subseteq \mathcal{P}} m(\mathcal{A}) = 1$
2. $\sum_{\mathcal{B} \subseteq \mathcal{A}, \ i \in \mathcal{B}} m(\mathcal{B}) \geq 0 \ \ \forall \mathcal{A} \subseteq \mathcal{P}, i \in \mathcal{P}$

All these conditions are satisfied:

- $m(\emptyset) = 0$ by definition
- $\sum_{\mathcal{A} \subseteq \mathcal{P}} m(\mathcal{A}) = \sum_{i=1}^{p} \lambda_i = 1$
- all $m(\mathcal{B})$ are non-negative as $\lambda_i \geq 0$

Moreover, as $m(\mathcal{A}) = 0 \ \forall \mathcal{A}$ such that $card(\mathcal{A}) > 2$, v is a 2-additive capacity. Therefore we have a capacity v and its set of Möbius coefficients such that

$\forall z \in \mathcal{Y}$,

$$\begin{aligned} f_v^C(y) &= \sum_{\mathcal{A} \subseteq \mathcal{P}} m(\mathcal{A}) \min_{i \in \mathcal{A}} y_i \\ &= \sum_{i \in \mathcal{P}} \lambda_i y_i \\ &\geq \sum_{i \in \mathcal{P}} \lambda_i p^\delta(z)_i \\ &\geq \sum_{\mathcal{A} \subseteq \mathcal{P}} m(\mathcal{A}) \min_{i \in \mathcal{A}} z_i \\ &\geq f_v^C(z) \end{aligned}$$

So $y \in \mathcal{Y}_{C^2}$. □

From this theorem, we can derive an algorithm to generate a set $\mathcal{X}_{C^2}$ containing solutions of a MOCO problem that optimize a 2-additive Choquet integral.

For all the permutations σ on $\mathcal{P}$, we have to:

1. Consider an application δ such that $\delta(1) = 1$ and $\delta(i) < i \ \forall i \neq 1$.
2. Determine the set $\mathcal{P}_\sigma^\delta(\mathcal{Y})$ containing the projections obtained with the application p_σ^δ for each $y \in \mathcal{Y}$.
3. Determine the solutions in O_σ that optimize a WS considering $\mathcal{P}_\sigma^\delta(\mathcal{Y})$.

4 Experiments

We have applied the algorithm for defined Pareto fronts, that is, a Pareto front is given, and the aim is to determine, among the P-non-dominated points, the 2-additive Choquet optimal points.

To generate Pareto fronts, we have applied a heuristic to multiobjective knapsack instances. We have used knapsack instances with random profits. The heuristic is an adaptation of the one presented in [16]. Note that the aim is only to generate a set of non-dominated points to experiment the sufficient condition.

The results are given in Table 1 for $p = 4$, $k = 2$, and 250 points, and in Table 2 for $p = 4$, $k = 2$, and 500 points.

We have considered all possible applications δ. We have also computed the exact number of 2-additive Choquet optimal solutions with a linear program: for each point of the Pareto front, we check if there exists a 2-additive capacity v such that the Choquet integral of this point is better that all the other points. Note that this method can be applied since we consider the particular case of a given Pareto front. For the problem with 250 points, 140 points optimize a 2-additive Choquet integral and for the problem with 500 points, 200 points optimize a 2-additive Choquet integral. We see that our method can only reach a subset of this set (since the method is only based on a sufficient condition). The number of 2-additive Choquet optimal points generated depends on the

Table 1. Random multiobjective knapsack instances (250 points, 140 are 2-additive Choquet optimal)

δ	#2C-Optimal
(1,1,1,1)	124
(1,1,1,2)	124
(1,1,1,3)	131
(1,1,2,1)	131
(1,1,2,2)	131
(1,1,2,3)	139

Table 2. Random multiobjective knapsack instances (500 points, 200 are 2-additive Choquet optimal)

δ	#2C-Optimal
(1,1,1,1)	164
(1,1,1,2)	164
(1,1,1,3)	174
(1,1,2,1)	186
(1,1,2,2)	187
(1,1,2,3)	196

application δ. For the set with 250 points, with $\delta = (1, 1, 1, 1)$, 124 points are generated, while with the application $(1, 1, 2, 3)$, 139 points are computed. For the set with 500 points, with $\delta = (1, 1, 1, 1)$, 164 points are generated, while with the application $(1, 1, 2, 3)$, 196 points are computed. Some application δ allows thus to reach more 2-additive Choquet optimal solutions. However, even by merging the sets obtained with all possible applications δ, they are still 2-additive Choquet optimal solutions that cannot be reached with our method based on the sufficient condition.

5 Conclusion

We have introduced in this paper a sufficient condition to produce 2-additive Choquet optimal solutions of multiobjective combinatorial optimization problems. We have also presented an algorithm to obtain these solutions based on this condition. The algorithm can be applied to generate an interesting subset of the Pareto optimal set (in case of the size of this set is too high). This work about generating 2-additive Choquet optimal solutions opens many new perspectives:

– As our condition is only sufficient, a necessary and sufficient condition will be required to generate all the 2-additive Choquet optimal solutions of a MOCO problem. The condition will also have to be generalized to $k > 2$.

– Following [17], it will be interesting to study and to define what brings exactly and concretely (for a decision maker) the 2-additive Choquet optimal solutions that are not WS optimal solutions, given that they are harder to compute.
– More experiments will be needed to show the differences between WS optimal solutions, Choquet optimal solutions and 2-additive Choquet optimal solutions of MOCO problems.

References

1. Ehrgott, M.: Multicriteria Optimization, 2nd edn. Springer, Berlin (2005)
2. Choquet, G.: Theory of capacities. Annales de l'Institut Fourier (5), 131–295 (1953)
3. Grabisch, M., Labreuche, C.: A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid. Annals OR 175(1), 247–286 (2010)
4. Grabisch, M., Marichal, J.C., Mesiar, R., Pap, E.: Aggregation functions. Encyclopedia of Mathematics and its Applications, vol. 127. Cambridge University Press, Cambridge (2009)
5. Yager, R.: On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Trans. Systems, Man and Cybern. 18, 183–190 (1998)
6. Torra, V.: The weighted OWA operator. Int. J. Intell. Syst. (12), 153–166 (1997)
7. Grabisch, M.: k-order additive discrete fuzzy measures and their representation. Fuzzy Sets and Systems (92), 167–189 (1997)
8. Fouchal, H., Gandibleux, X., Lehuédé, F.: Preferred solutions computed with a label setting algorithm based on Choquet integral for multi-objective shortest paths. In: 2011 IEEE Symposium on Computational Intelligence in Multicriteria Decision-Making (MDCM), pp. 143–150. IEEE, Paris (2011)
9. Galand, L., Perny, P., Spanjaard, O.: Choquet-based optimisation in multiobjective shortest path and spanning tree problems. European Journal of Operational Research 204(2), 303–315 (2010)
10. Galand, L., Perny, P., Spanjaard, O.: A branch and bound algorithm for Choquet optimization in multicriteria problems. Proceedings Lecture Notes in Economics and Mathematical Systems 634, 355–365 (2011)
11. Lust, T., Rolland, A.: Choquet optimal set in biobjective combinatorial optimization. Computers & Operations Research 40(10), 2260–2269 (2013)
12. Lust, T., Rolland, A.: On the computation of Choquet optimal solutions in multicriteria decision contexts. In: Ramanna, S., Lingras, P., Sombattheera, C., Krishna, A. (eds.) MIWAI 2013. LNCS, vol. 8271, pp. 131–142. Springer, Heidelberg (2013)
13. Grabisch, M.: The application of fuzzy integrals in multicriteria decision making. European Journal of Operational Research 89, 445–456 (1996)
14. Rota, G.C.: On the foundations of combinatorial theory I. Theory of Mobius functions. Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 2, 340–368 (1964)
15. Chateauneuf, A., Jaffray, J.Y.: Some characterizations of lower probabilities and other monotone capacities through the use of mobius inversion. Mathematical Social Sciences 17(3), 263–283 (1989)
16. Lust, T., Teghem, J.: The multiobjective multidimensional knapsack problem: a survey and a new approach. International Transactions in Operational Research 19(4), 495–520 (2012)
17. Meyer, P., Pirlot, M.: On the expressiveness of the additive value function and the Choquet integral models. In: DA2PL 2012: from Multiple Criteria Decision Aid to Preference Learning, pp. 48–56 (2012)