Subject

In this tutorial, we show how to use the **FORWARD ENTRY REGRESSION** component: it performs a multiple linear regression with a forward variable selection based on partial correlation.

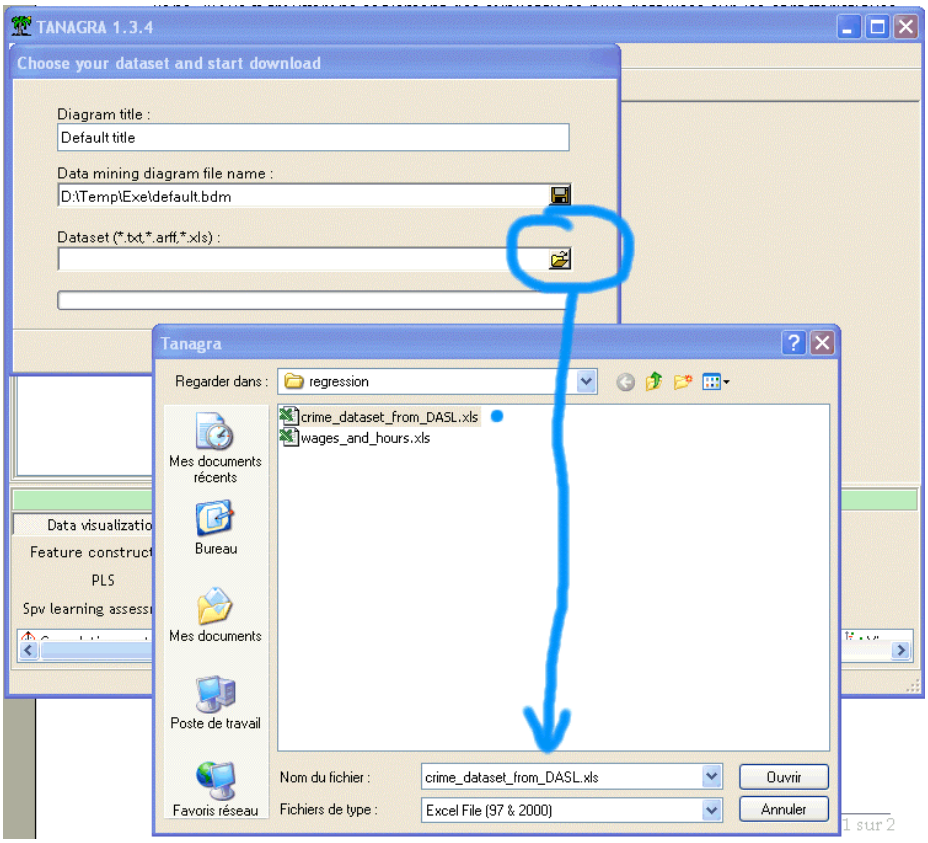
Dataset

We use CRIME_DATASET_FROM_DASL.XL from the DASL website[1]. It contains various characteristics of 47 states of USA. We want to explain the criminality from unemployment, education level, …

Forward Selection for Regression

Import the dataset

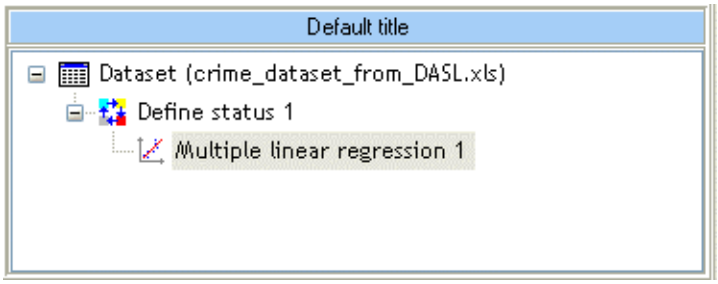
We build a new diagram and import the dataset with the FILE / NEW menu.



[1] http://lib.stat.cmu.edu/DASL/Stories/USCrime.html

Linear multiple regression

First, we want to perform a regression with the whole variables. We add a DEFINE STATUS component in the diagram and set CRIME RATE as TARGET, and the other variables as INPUT. We add the LINEAR MULTIPLE REGRESSION component.



We obtain the following results.

Global results

Endogenous attribute	**CrimeRate**
Examples	47
R^2	0.769236
Adjusted-R^2	0.678329
Sigma error	21.935649
F-Test (13,33)	8.4618 (0.000000)

Analysis of variance

Source	xSS	d.f.	xMS	F	p-value
Regression	52930.5756	13	4071.5827	8.4618	0.0000
Residual	15878.6992	33	481.1727		
Total	68809.2747	46			

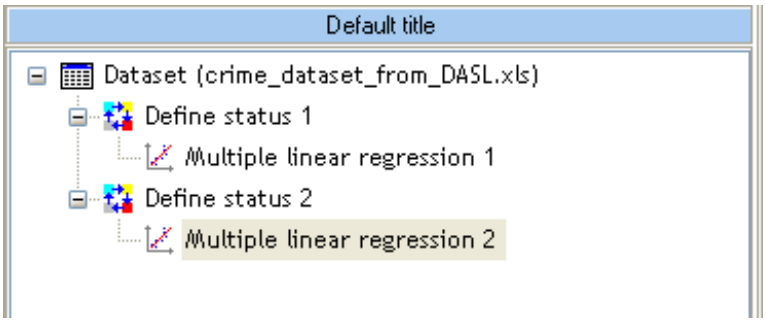
Coefficients

Attribute	Coef.	std	t(33)	p-value
Constant	-691.837589	155.887910	-4.438045	0.000096
Male14-24	1.039810	0.422708	2.459875	0.019306
Southern	-8.308312	14.911587	-0.557172	0.581170
Education	1.801601	0.649650	2.773186	0.009060
Expend60	1.607818	1.058667	1.518720	0.138357
Expend59	-0.667258	1.148773	-0.580844	0.565292
Labor	-0.041031	0.153477	-0.267344	0.790868
Male	0.164795	0.209932	0.784993	0.438057
PopSize	-0.041277	0.129516	-0.318701	0.751962
NonWhite	0.007175	0.063867	0.112338	0.911236
Unemp14-24	-0.601675	0.437154	-1.376345	0.177983
Unemp35-39	1.792263	0.856111	2.093493	0.044069
FamIncome	0.137358	0.105830	1.297913	0.203316
IncUnderMed	0.792933	0.235085	3.372959	0.001913

The results seem encouraging -- $R^2 = 0.77$ -- and 4 variables are significant -- p-value lower than 0.05: MALE14-24, EDUCATION, UNEMP35-39, and INCUNDERMED.

Regression with the significant exogenous variables

We want to perform a new regression with only the significant variables. We add a DEFINE STATUS component and set as INPUT the significant variables above, TARGET is always CRIMERATE. We add a new REGRESSION component.



The results are particularly disappointing!

Global results

Endogenous attribute	**CrimeRate**
Examples	47
R^2	0.229784
Adjusted-R^2	0.156430
Sigma error	35.522631
F-Test (4,42)	3.1325 (0.024221)

Analysis of variance

Source	xSS	d.f.	xMS	F	p-value
Regression	15811.2675	4	3952.8169	3.1325	0.0242
Residual	52998.0072	42	1261.8573		
Total	68809.2747	46			

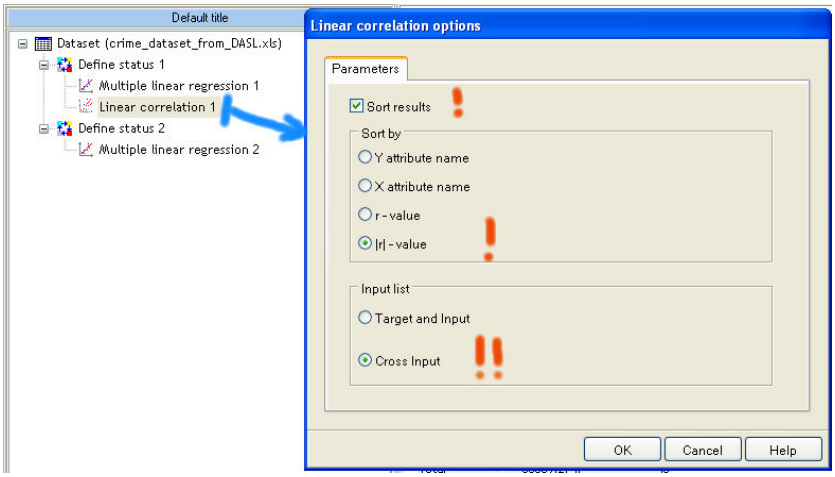
Coefficients

Attribute	Coef.	std	t(42)	p-value
Constant	-349.158324	155.025802	-2.252259	0.029592
Male14-24	0.767473	0.587023	1.307399	0.198189
Education	2.299542	0.789091	2.914165	0.005695
Unemp35-39	1.736663	0.706527	2.458027	0.018178
IncUnderMed	0.161780	0.227497	0.711130	0.480934

Explained variance is significantly lower and only two variables seems relevant: EDUCATION andUNEMP35-39. These results are not at all coherent with what we saw previously.

Correlation between the exogenous variables

We suspect a problem of colinearity between the exogenous variables. In order to check that, we add the LINEAR CORREALTION in the diagram and we set the following parameters.



We see that several variables are highly correlated. In some cases, the square of the correlation coefficient is higher than the R^2 of the regression.

Linear correlation 1

Parameters

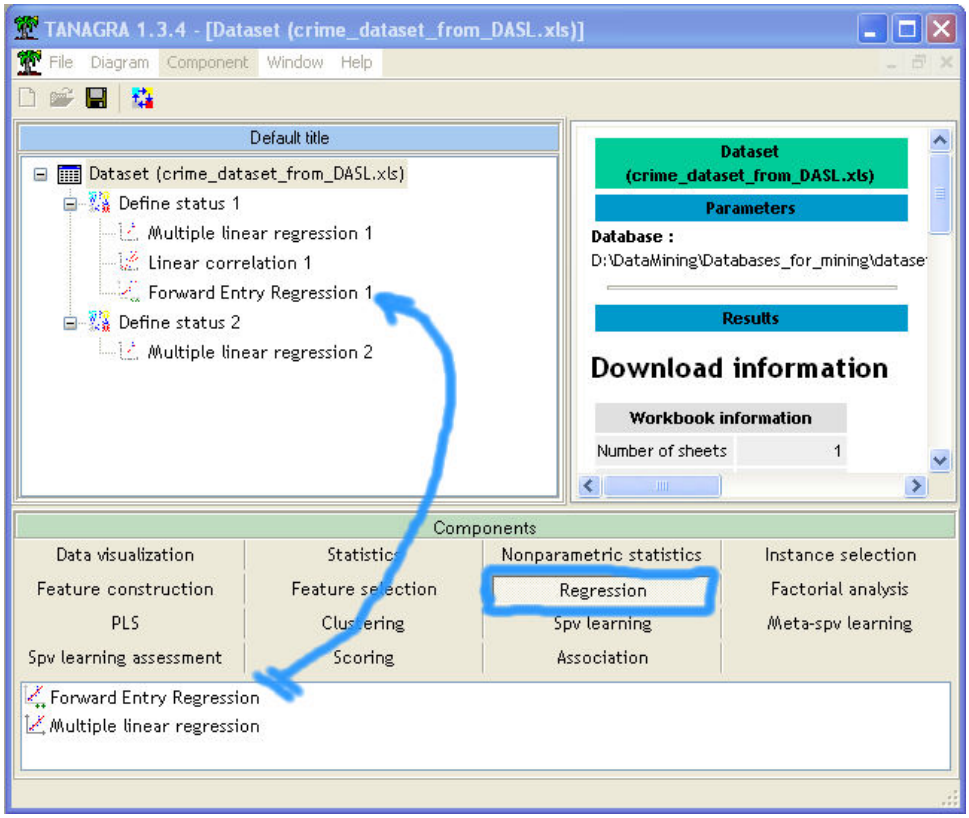
Cross-tab parameters	
Sort results	yes
Sort criterion	|r| statistic
Input list	Cross-input (Y x X)

Results

Y	X	r	r^2	t	Pr(>|t|)
Expend60	Expend59	0.9936	0.9872	58.9449	0.0000
FamIncome	IncUnderMed	-0.8840	0.7815	-12.6848	0.0000
Expend59	FamIncome	0.7943	0.6309	8.7694	0.0000
Expend60	FamIncome	0.7872	0.6197	8.5636	0.0000
Education	IncUnderMed	-0.7687	0.5908	-8.0610	0.0000
Southern	NonWhite	0.7671	0.5884	8.0213	0.0000
Unemp14-24	Unemp35-39	0.7459	0.5564	7.5129	0.0000
Southern	IncUnderMed	0.7372	0.5434	7.3186	0.0000
Education	FamIncome	0.7360	0.5417	7.2930	0.0000
Southern	Education	-0.7027	0.4938	-6.6261	0.0000
NonWhite	IncUnderMed	0.6773	0.4588	6.1759	0.0000

Forward selection for regression

There are various solutions for colinearity problem in the regression. The FORWARD ENTRY REGRESSION performs a forward selection of variables using the partial correlation measurements.



The quality of the regression with 5 variables is close to the first regression with the whole dataset: irrelevant variables are rejected.

Global results

Endogenous attribute	**CrimeRate**
Examples	47
R^2	0.729635
Adjusted-R^2	0.696663
Sigma error	21.301348
F-Test (5,41)	22.1293 (0.000000)

Analysis of variance

Source	xSS	d.f.	xMS	F	p-value
Regression	50205.6311	5	10041.1262	22.1293	0.0000
Residual	18603.6437	41	453.7474		
Total	68809.2747	46			

Coefficients

Attribute	Coef.	std	t(41)	p-value
Constant	-524.374333	95.115565	-5.513023	0.000002
Expend60	1.233122	0.141635	8.706359	0.000000
IncUnderMed	0.634926	0.146846	4.323752	0.000096
Education	2.030773	0.474189	4.282623	0.000109
Male14-24	1.019822	0.353203	2.887356	0.006175
Unemp35-39	0.913608	0.434092	2.104642	0.041496

Below, TANAGRA shows the detailed results and the steps of the computation.

Forward Selection Process

partial corr. F (p-value)	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
d.f.	45	44	43	42	41	40
r(Y,Xj*/Xj1,Xj2...)	Expend60 : 0.6876	IncUnderMed : 0.4516	Education : 0.4509	Male14-24 : 0.3226	Unemp35-39 : 0.3123	-
R^2	0.4728	0.5803	0.6656	0.7004	0.7296	-
Male14-24	-0.0895 0.36 (0.5498)	0.4123 9.01 (0.0044)	0.2505 2.88 (0.0970)	0.3226 4.88 (0.0327)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)
Southern	-0.0906 0.37 (0.5446)	0.2458 2.83 (0.0997)	-0.1081 0.51 (0.4797)	0.0424 0.08 (0.7847)	-0.0393 0.06 (0.8023)	-0.0489 0.10 (0.7586)
Education	0.3228 5.24 (0.0269)	-0.0145 0.01 (0.9236)	0.4509 10.97 (0.0019)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)
Expend60	0.6876 40.36 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)
Expend59	0.6667 36.01 (0.0000)	-0.2007 1.85 (0.1810)	-0.1031 0.46 (0.5005)	-0.1360 0.79 (0.3786)	-0.1484 0.92 (0.3423)	-0.1285 0.67 (0.4172)
Labor	0.1889 1.66 (0.2036)	0.1461 0.96 (0.3325)	0.3004 4.26 (0.0450)	0.0562 0.13 (0.7173)	0.0381 0.06 (0.8085)	0.1501 0.92 (0.3428)
Male	0.2139 2.16 (0.1488)	0.2628 3.26 (0.0777)	0.3967 8.03 (0.0070)	0.2255 2.25 (0.1410)	0.1900 1.54 (0.2224)	0.1135 0.52 (0.4743)
PopSize	0.3375 5.78 (0.0204)	-0.0395 0.07 (0.7943)	-0.2125 2.03 (0.1611)	-0.1307 0.73 (0.3977)	-0.0627 0.16 (0.6896)	-0.0734 0.22 (0.6440)
NonWhite	0.0326 0.05 (0.8278)	0.2531 3.01 (0.0896)	-0.1123 0.55 (0.4625)	0.0428 0.08 (0.7828)	-0.0894 0.33 (0.5685)	-0.0988 0.39 (0.5335)
Unemp14-24	-0.0505 0.11 (0.7362)	-0.0282 0.03 (0.8526)	0.0283 0.03 (0.8537)	0.0566 0.13 (0.7153)	0.1570 1.04 (0.3146)	-0.1861 1.44 (0.2380)
Unemp35-39	0.1773 1.46 (0.2331)	0.0701 0.22 (0.6432)	-0.0094 0.00 (0.9513)	0.1643 1.17 (0.2865)	0.3123 4.43 (0.0415)	0.0000 0.00 (0.0000)
FamIncome	0.4413 10.88 (0.0019)	-0.2233 2.31 (0.1358)	0.2722 3.44 (0.0705)	0.1859 1.50 (0.2269)	0.2815 3.53 (0.0674)	0.2595 2.89 (0.0970)
IncUnderMed	-0.1790 1.49 (0.2286)	0.4516 11.27 (0.0016)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)	0.0000 0.00 (0.0000)

At the first step, EXPEND60 is the most correlated with the endogenous variable (r = 0.6876). With the t of Student (the Fisher's F is the square of the Student's t), we see that this correlation is significant (at the significance level of 0.05).

At the second step, we compute the correlation between the endogenous the resulting variables, by removing the information given by EXPEND60: this is a partial correlation. We see that INCUNDERMED is the most correlated with the endogenous and it is highly significant (the degree of freedom of the test has been modified!).

We continue the process until we cannot introduce anymore a new variable: we obtain 5 relevant variables.

On the DASL website, the authors propose the same regression -- http://lib.stat.cmu.edu/DASL/Stories/USCrime.html.

Dependent variable is: **R**
No Selector
48 total cases of which 1 is missing
R squared = 73.0% R squared (adjusted) = 69.7%
s = 21.30 with 47 - 6 = 41 degrees of freedom

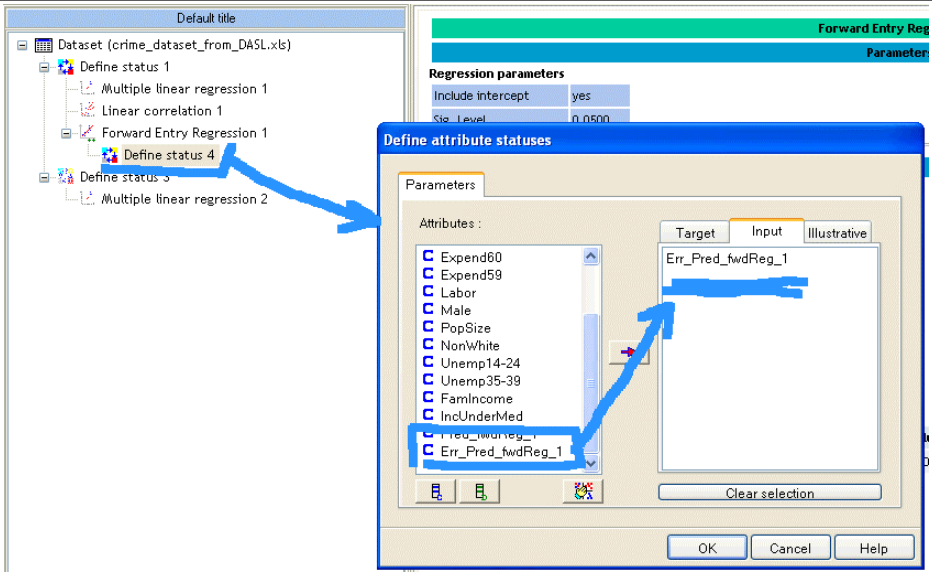
Source	Sum of Squares	df	Mean Square	F-ratio
Regression	50205.6	5	10041.1	22.1
Residual	18603.6	41	453.747	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	-524.374	95.12	-5.51	≤ 0.0001
Age	1.01982	0.3532	2.89	0.0062
Ed	2.03077	0.4742	4.28	0.0001
U2	0.913608	0.4341	2.10	0.0415
X	0.634926	0.1468	4.32	≤ 0.0001
Ex0	1.23312	0.1416	8.71	≤ 0.0001

Normality test

The regression component produces two variables: the prediction and the residuals. In order to check the validity of the regression, it is possible to test the normality of the residuals.

We add a new DEFINE STATUS in the diagram and set the variable ERR_PRED_LMREG_2 as INPUT.



We add the NORMALITY TEST component. At the significance level of 5%, we see that the observed residuals are compatible with the assumption of normality.